April 21, 2021 3:05 PM

1 Setting

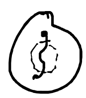
X is a manifold $\dim X \geq 3$.

dim 2

if $\phi : [0,1] \to X$

$X \setminus \phi[0,1]$

is still connected



dim ≥ 3.

IF $\phi : [0,1] \to X$

$U \underset{\text{open}}{\subseteq} X$

$U \setminus \phi[0,1]$

is still connected

SRI

$G = \mathrm{Homeo}(X) \geq H$ "locally transitive"

- Is UMF metrizable?

M, nVT, T

If G polish group

$\{Y :$ minimal flows$\}$ ~ Polish space

If UMF is metrizable

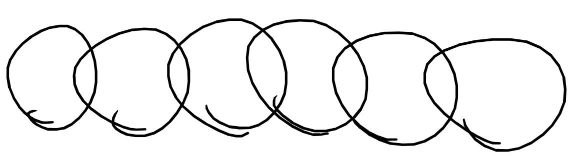
1.1.

G-action → minimal flow

$C(X) = \{ c \mid K \in c \quad K \subseteq X$ connected compact$\}$

totally ordered c

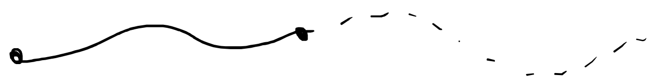
- Rays / Tubes

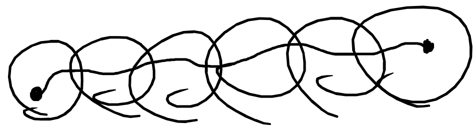


(Basic open sets,

– Rays $\quad \phi : [0,\infty) \to X \quad$ continuous injective dense.

$$C_\phi = \{\phi[0,r]\} \cup \{X\}$$





2 The Plan

Criterion Rosendal.

If G polish, Y minimal flow

Y has only meager orbits

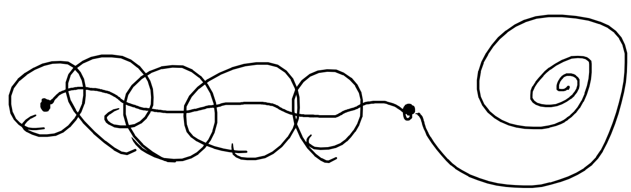
Iff $\exists \varepsilon, \forall V \subseteq Y, \exists W_1, W_2 \underset{open}{\subseteq} V, \quad G_\varepsilon \cdot W_1 \cap W_2 = \emptyset$

$$G_\varepsilon = \{ g \in G \mid \underline{d_G(1,g)} < \varepsilon \}$$

Let d be a metric on X.

$$d(g,h) = \sup_{x \in X} \{ d(gx,hx) \}$$

$V =$ tube



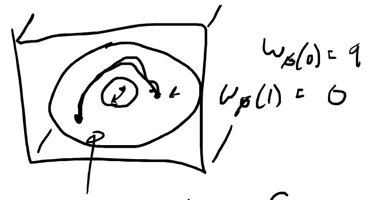
[BYMT] If G is a polish with metrizable UMF, any minimal flow has a comeager orbit.

Winding in knots.

3] Fix any chart $Z \cong [-1,1]^n$, $\quad \varepsilon$ small enough.

$$A = \left\{ \frac{1}{4} \le x_1^2 + x_2^2 \le \frac{9}{16} \right\} \times [-1,1]^{n-2}$$

$$A = \left\{ \frac{1}{4} \le x_1^2 + x_2 \le \frac{1}{16} \right\} \times [-1,1]$$



$W(\phi) = -9$.

$$\begin{tikzcd} {[0,1]} \arrow[r, "\phi"] \arrow[drr, "w_\phi"'] & A \arrow[r] & \mathbb{T} \\ & & \mathbb{R} \arrow[u] \end{tikzcd}$$

$$W(\phi) = w_\phi(1) - w_\phi(0)$$

<u>Lemma</u> If $\phi\, [0,1] \to A$ is an arc $g \in G_\varepsilon$ $g \cdot \mathrm{Im}(\phi) \subseteq A$.

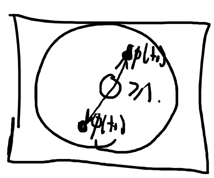
$$|W(\phi) - W(g\cdot\phi)| < 1.$$

$\phi(0)$ · $\quad g \cdot \phi(0)$

$$|w_\phi(0) - w_{g\phi}(0)| < 1/2.$$

$\forall t \in [0,1]$ $\quad |W_\phi(t) - W_{g\phi}(t)| < 1/2.$

If not, one part t_0 $\quad |W_\phi(t_0) - W_{g\phi}(t_0)| = 1/2.$



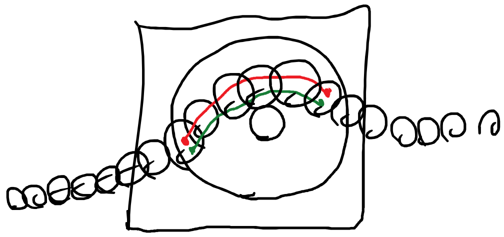
We suppose $d(g,1) < \varepsilon < 1$.

$$|W(\phi) - W(g\phi)| \le |w_\phi(1) - w_{g\phi}(1)| + |w_\phi(0) - w_{g\phi}(0)| < 1.$$

<u>Lemma</u> If $U_0, \dots, U_n$ is a tube (assume U_i are balls of radius ε).

..

and $\phi, \psi : [0,1] \to A$ with $U_i, \dots, U_j$

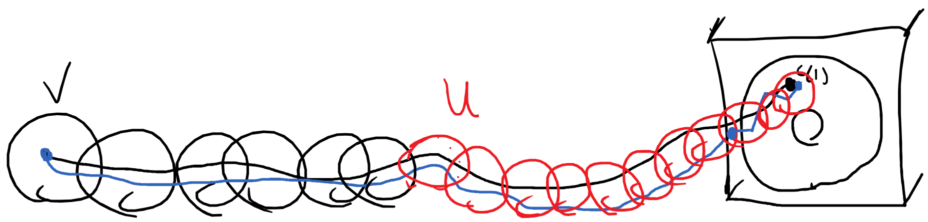


$$|w(\phi) - w(\psi)| < 1$$

$$|w(\phi)| \leq |i-j| + 1.$$

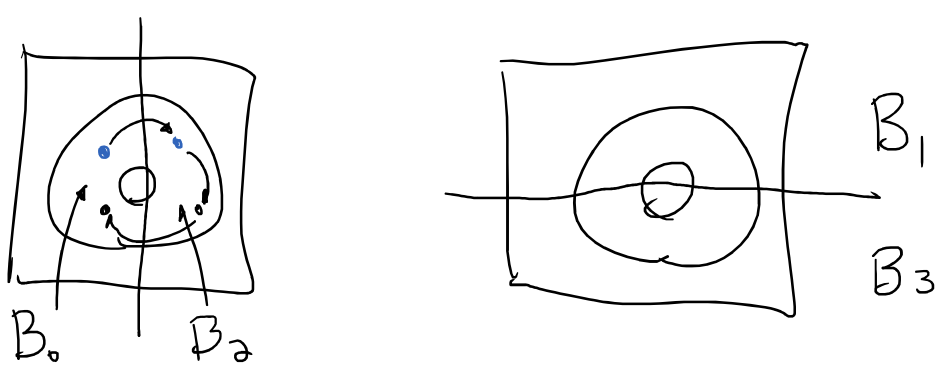
4 Criterion of Rondal.

Let V a tube.



Pick a ray $c \in V$. Every ray c' extending $c|_{[0,1]}$ are in V.
$c(1) \in \mathrm{Int}(A)$

Pick an arc $\phi : [0,1] \to X$ compatible with U



$$B_0 \cap B_1 \longrightarrow B_1 \cap B_2 \to B_2 \cap B_3 \longrightarrow B_3 \cap B_0$$

$Y_0 = \varphi(1)$ Y_1 Y_2 .... Y_N (N is large)

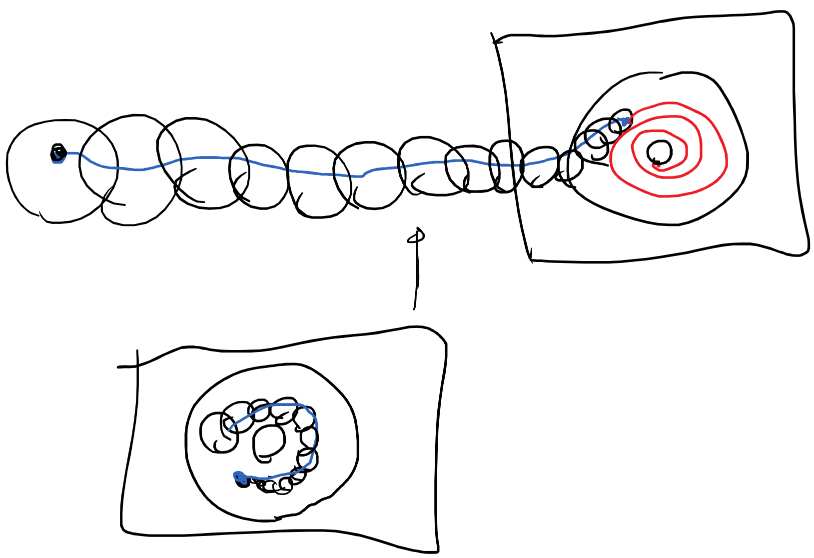
$Y_i = \phi^*(i+1)$

$Y_i \longrightarrow Y_{i+1}$ need to avoid $\phi^*[0, i+1]$

Stay within $A_\phi \setminus \phi^*[0, i+1]$

$\boxed{\text{SRI}}$

$\phi \to \phi_\circlearrowleft$, $\phi \to \phi_\circlearrowright$



Not hard to blow up $\phi_\circlearrowleft$ into $U_\circlearrowleft$

$\phi_\circlearrowright$ into $U_\circlearrowright$

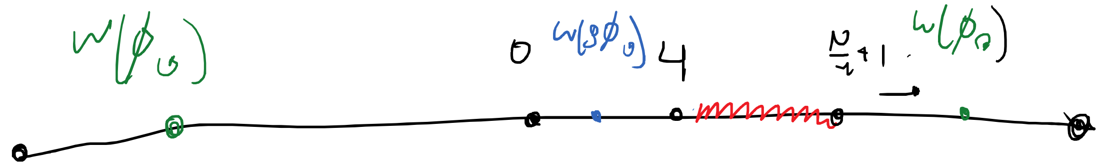
If ϕ is compatible with $U_\circlearrowleft$ (except the part outside of A)

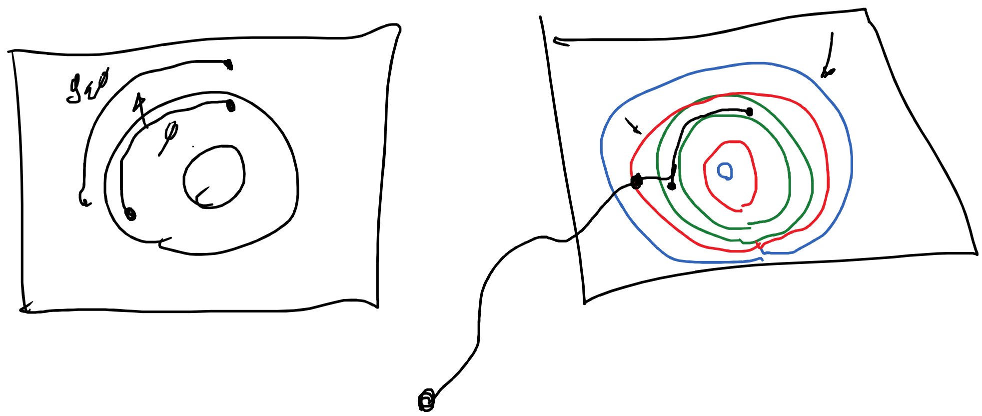
$w(\phi) \geq \frac{N}{n} - 1.$

$w(\phi) \geq \frac{N}{4} - 1$.

↳ If $g \in G_\epsilon$,

$w(g_\epsilon) \geq -4$.





$$W_1 = U_0 \qquad W_2 = U_2$$

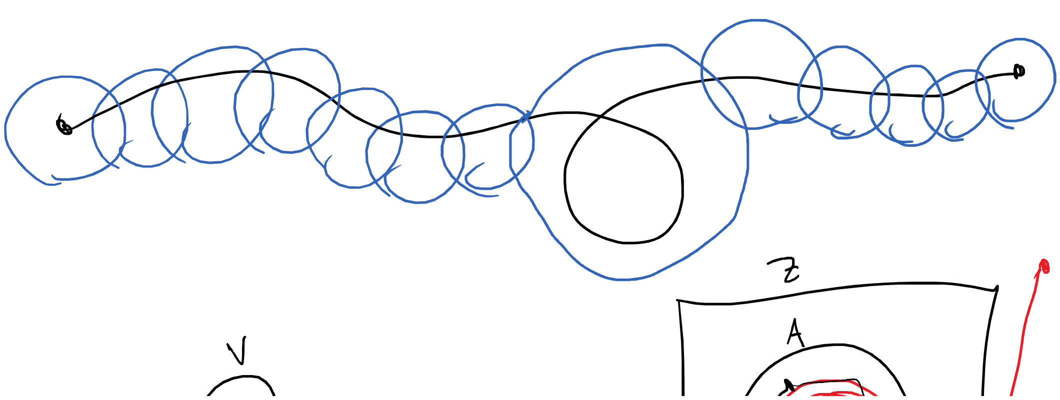
$$G_\epsilon W_1 \cap W_2 = \emptyset.$$

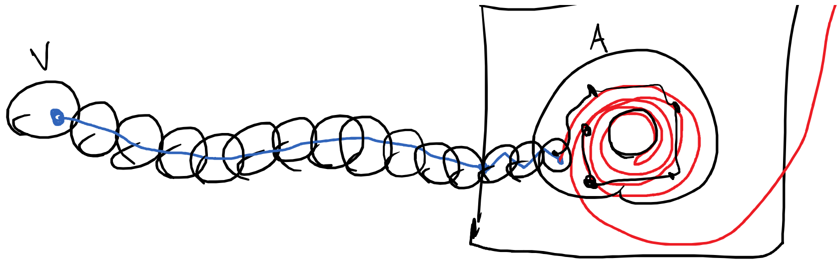
This works for every V.

So Rosendal's criteria.

Thus. $G \curvearrowright C(X)$ with meager orbits

Thus by BYMT, $M(G)$ is non metrizable.




V
A